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A NEW GAIN MATRIX OF THE ROTOR FLUX LUENBERGER OBSERVER FOR INDUCTION MOTORS – ROTATING METHOD OF THE EIGENVALUES

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Abstract: In this article presents a new gain matrix of the rotor flux Luenberger observer obtained based on the rotating method of the eigenvalues. In order to determine the gain matrix of the Luenberger observer, the mathematical model of the induction motor with iron loss, is used. The Luenberger observer estimates the position and modulus of the rotor flux space vector. The validation of the new gain matrix is done by numerical simulation in Matlab-Simulink.

Key words: induction motors, numerical simulation, Luenberger observer.

1. INTRODUCTION

In present, direct rotor field-oriented control (RDFOC) of induction motors, require estimation of the position and modulus of the rotor flux space vector [1] – [3].

The dynamic performances of vector control systems are closely linked of the quality of estimating the position and modulus of the rotor flux space vector [1] – [3].

In order to increase the quality of the estimation of the position of the rotor flux space vector, of the induction motor mathematical model with iron core losses is used [4], in the design of the gain matrix of the Luenberger observer. In this sense, the method of rotating the eigenvalues, proposed by R. Maceratini and G. Barba is used [5].

The essential contribution of this article is related to the presentation of a new gain matrix of the rotor flux Luenberger observer. In the final part of the article, the dynamic performances of the Luenberger observer, based on the new gain matrix, are evidenced by numerical simulation in Matlab-Simulink.

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2. LUENBERGER OBSERVER

For a better understanding of the method for determining the new gain matrix of the Luenberger observer, in the following are presentation two mathematical models of the induction motor.

- *the stator currents-rotor fluxes model* (in which the iron losses are neglected) [1]:

$$\frac{d}{dt}\begin{bmatrix} \underline{i}_s \\ \underline{\psi}_r \end{bmatrix} = \begin{bmatrix} a_{11} & a_{13} - j \cdot a_{14} \cdot z_p \cdot \omega_r \\ a_{31} & a_{33} + j \cdot z_p \cdot \omega_r \end{bmatrix} \cdot \begin{bmatrix} \underline{i}_s \\ \underline{\psi}_r \end{bmatrix} + \begin{bmatrix} b_{11} \\ 0 \end{bmatrix} \cdot \underline{u}_s \qquad (1)$$

where $\underline{i}_s = i_{ds} + j \cdot i_{qs}$; $\underline{\psi}_r = \psi_{dr} + j \cdot \psi_{qr}$; $\underline{u}_s = u_{ds} + j \cdot u_{qs}$; $j = \sqrt{-1}$;

$$a_{11} = -\left(\frac{1}{T_s \cdot \sigma} + \frac{1-\sigma}{T_r \cdot \sigma}\right);\ a_{13} = \frac{L_m}{L_s \cdot L_r \cdot T_r \cdot \sigma};\ a_{14} = \frac{L_m}{L_s \cdot L_r \cdot \sigma};\ a_{31} = \frac{L_m}{T_r};\ a_{33} = -\frac{1}{T_r};$$

$$b_{11} = \frac{1}{L_s \cdot \sigma}; T_s = \frac{L_s}{R_s}; T_r = \frac{L_r}{R_r}; \sigma = 1 - \frac{L_m^2}{L_s \cdot L_r}$$

- *the stator currents-rotor fluxes-air-gap fluxes model* (the parallel model in which iron losses are taken into account) [4], [6]:

$$\frac{d}{dt}\begin{bmatrix} \underline{i}_s \\ \underline{\psi}_r \\ \underline{\psi}_m \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{13} & \alpha_{15} \\ 0 & \alpha_{33} + j \cdot z_p \cdot \omega_r & -\alpha_{33} \\ \alpha_{51} & \alpha_{53} & \alpha_{55} \end{bmatrix} \cdot \begin{bmatrix} \underline{i}_s \\ \underline{\psi}_r \\ \underline{\psi}_m \end{bmatrix} + \begin{bmatrix} \beta_{11} \\ 0 \\ 0 \end{bmatrix} \cdot \underline{u}_s \qquad (2)$$

where $\underline{i}_s = i_{ds} + j \cdot i_{qs}$; $\underline{\psi}_r = \psi_{dr} + j \cdot \psi_{qr}$; $\underline{\psi}_m = \psi_{dm} + j \cdot \psi_{qm}$; $\underline{u}_s = u_{ds} + j \cdot u_{qs}$; $j = \sqrt{-1}$;

$$\alpha_{11} = -\frac{R_s + R_f}{L_{\sigma s}};\ \alpha_{13} = -\frac{R_f}{L_{\sigma s} \cdot L_{\sigma r}}; \alpha_{15} = R_f \cdot \frac{L_r}{L_{\sigma s} \cdot L_{\sigma r} \cdot L_m}; \alpha_{33} = -\frac{R_r}{L_{\sigma r}};\ \alpha_{51} = R_f;$$

$$\alpha_{53} = \frac{R_f}{L_{\sigma r}};\ \alpha_{55} = -R_f \cdot \frac{L_r}{L_{\sigma r} \cdot L_m};\ \beta_{11} = \frac{1}{L_{\sigma s}};\ L_r = L_{\sigma r} + L_m;\ L_s = L_{\sigma s} + L_m$$

The following notations were used in the mathematical models presented above: R_r, R_s – rotor/stator resistances; L_r, L_s– rotor/stator inductances; L_m – mutual inductance; R_f – iron loss resistance; T_r, T_s – rotor/stator time-constants; σ – leakage factor; z_p – number of pole pairs; ω_r – mechanical angular speed; $\underline{u}_s$ – stator voltage space vector; $\underline{i}_s$ – stator current space vector; $\underline{\psi}_r$ – rotor flux space vector; $\underline{\psi}_m$ – space vector air-gap flux.

Under these conditions, the relations that define the Luenberger observer are [6]:

$$\frac{d}{dt}\begin{bmatrix}\hat{\underline{i}}_s \\ \hat{\underline{\psi}}_r\end{bmatrix} = \begin{bmatrix} a_{11} & a_{13} - j\cdot a_{14}\cdot z_p\cdot\omega_r \\ a_{31} & a_{33} + j\cdot z_p\cdot\omega_r \end{bmatrix}\cdot\begin{bmatrix}\hat{\underline{i}}_s \\ \hat{\underline{\psi}}_r\end{bmatrix} + \begin{bmatrix} b_{11} \\ 0\end{bmatrix}\cdot\underline{u}_s + G\cdot C\cdot\left(\begin{bmatrix}\underline{i}_s \\ \underline{\psi}_r\end{bmatrix} - \begin{bmatrix}\hat{\underline{i}}_s \\ \hat{\underline{\psi}}_r\end{bmatrix}\right) \tag{3}$$

where the estimated variables are denoted by "^"; $C = \begin{bmatrix}1 & 0\end{bmatrix}$ and G is gain matrix Luenberger

$$G = \begin{bmatrix} g_{11} + j\cdot g_{12} \\ g_{21} + j\cdot g_{22}\end{bmatrix} \tag{4}$$

To determine the elements of the matrix G, in the following we will define the following matrix

$$\Gamma = \begin{bmatrix} \alpha_{11} & \alpha_{13} \\ 0 & \alpha_{33} + j\cdot z_p\cdot\omega_r \end{bmatrix} \tag{5}$$

From the above relationship, we notice that Γ is a submatrix of the state matrix (first 2 rows and 2 columns), from the component of the mathematical model defined by the relation (2).

Following the tests, it observed that the spectrum of eigenvalues of the matrix Γ is located to the left of the complex plane. The spectrum of the eigenvalues of the Γ matrix is obtained from the following relation

$$\det\left[\lambda_m\cdot I_2 - \Gamma\right] = 0 \tag{6}$$

where I_2 is the second order unit matrix and $\lambda_m = \lambda_{m1} + j\cdot\lambda_{m2}$ is the spectrum of eigenvalues of the matrix Γ.

Under these conditions, the elements of the gain matrix (G) are determined from the following imposed relation

$$\lambda_e = k_L\cdot\lambda_m;\quad k \in £ \tag{7}$$

where: $\lambda_e = \lambda_{e1} + j\cdot\lambda_{e2}$ is the spectrum of the Luenberger observer eigenvalues; $k_L = k_1 + j\cdot k_2$ is the proportionality coefficient; $£$ is the set of complex numbers.

The relation of determining the spectrum of the Luenberger observer eigenvalues is

$$\det\left[\lambda_e\cdot I_2 - \left(A - G\cdot C\right)\right] = 0 \tag{8}$$

where $A = \begin{bmatrix} a_{11} & a_{13} - j\cdot a_{14}\cdot z_p\cdot\omega_r \\ a_{31} & a_{33} + j\cdot z_p\cdot\omega_r \end{bmatrix}$.

Taking into account the relation (7), following the calculations we obtain the elements of the matrix G.

$$g_{11} = a_{11} + a_{33} - k_1 \cdot \left(\alpha_{11} + \alpha_{33}\right) + k_2 \cdot z_p \cdot \omega_r \tag{9}$$

$$g_{12} = z_p \cdot \omega_r \cdot \left(1 - k_1\right) - k_2 \cdot \left(\alpha_{11} + \alpha_{33}\right) \tag{10}$$

$$g_{21} = a_{31} + \frac{a_{11} - g_{11}}{a_{14}} + \frac{\alpha_{11}}{a_{14}} \frac{\left(k_2^2 - k_1^2\right)\left(z_p^2 \cdot \omega_r^2 + \alpha_{33} \cdot a_{33}\right) + 2 \cdot k_1 \cdot k_2 \cdot z_p \cdot \omega_r \left(a_{33} - \alpha_{33}\right)}{z_p^2 \cdot \omega_r^2 + a_{33}^2} \tag{11}$$

$$g_{22} = \frac{\alpha_{11}}{a_{14}} \cdot \frac{\left(k_2^2 - k_1^2\right) \cdot z_p \cdot \omega_r \cdot \left(a_{33} - \alpha_{33}\right) - 2 \cdot k_1 \cdot k_2 \cdot \left(z_p^2 \cdot \omega_r^2 + \alpha_{33} \cdot a_{33}\right)}{z_p^2 \cdot \omega_r^2 + a_{33}^2} - \frac{g_{12}}{a_{14}} \tag{12}$$

If in the above relations, we impose $k_2 = 0$ and $k_1 > 1$, we obtain the elements of the matrix G based on the method of H. Kubota (proportional eigenvalues method) [7]. The elements of the matrix G in this case are identical to those proposed by O.Stoicuta [6].

On the other hand, if $k_1 = k \cdot \cos\left(\theta\right)$ and $k_2 = k \cdot \sin\left(\theta\right)$, the matrix G e is obtained based on the method proposed by R. Maceratini and G. Barba (method of rotating the eigenvalues) [5].

In this case, the proportionality coefficient from relation (7) becomes [5]:

$$k_L = k_1 + j \cdot k_2 = k \cdot e^{j \cdot \theta};\ 0 < \theta \le \pi/4\ ;\ k > 1 \tag{13}$$

From the above relation, it is observed that the method proposed by R. Maceratini and G. Barba is identical with the method proposed by H. Kubota, with the difference that simultaneously with the amplification of the eigenvalues by means of the proportionality coefficient k, there is also a rotation of the eigenvalues with an θ angle.

In the above relation, the θ angle can be chosen either constant or depending on the mechanical angular speed. (V. Bostan proposed this idea [8])

$$k_L = k_1 + j \cdot k_2 = k \cdot e^{j \cdot \theta};\ \theta = k_m \cdot \omega_r\ ;\ k > 1 \tag{14}$$

where $k_m = \theta_{\min} / \omega_{r_{\max}}$.

If in relation (7), we choose $\Gamma = A$, we obtain the elements of matrix proposed by G. R. Maceratini and G. Barba [5]. The matrix G in this case is defined by the following elements [5]

$$g_{11} = \left(1 - k_1\right) \cdot \left(a_{11} + a_{33}\right) + k_2 \cdot z_p \cdot \omega_r \tag{15}$$

$$g_{12} = (1 - k_1) \cdot z_p \cdot \omega_r - k_2 \cdot (a_{11} + a_{33}) \tag{16}$$

$$g_{21} = (1 - k_1^2 + k_2^2) \cdot (a_{31} + \gamma \cdot a_{11}) - \gamma \cdot g_{11} \tag{17}$$

$$g_{22} = -2 \cdot k_1 \cdot k_2 \cdot (a_{31} + \gamma \cdot a_{11}) - \gamma \cdot g_{12} \tag{18}$$

where $\gamma = 1/a_{14}$.

On the other hand, if in the above relations we impose $k_2 = 0$ and $k_1 > 1$, we obtain the elements of the matrix G, proposed by H. Kubota [7].

3. SIMULATION RESULTS

The validation of the gain matrix Luenberger - proposed (defined by relations (9) - (12)), is done by numerical simulation of the Luenberger observer in Matlab-Simulink. In this sense, a 1.5[kW] induction motor is used [6], [9]. The Luenberger observer is tested in open loop. In the numerical simulation, the induction motor starts based on the DOL (Direct On-Line) method, in load ($T_L = 10[N \cdot m]$).

The simulation results are presented in the following figure.

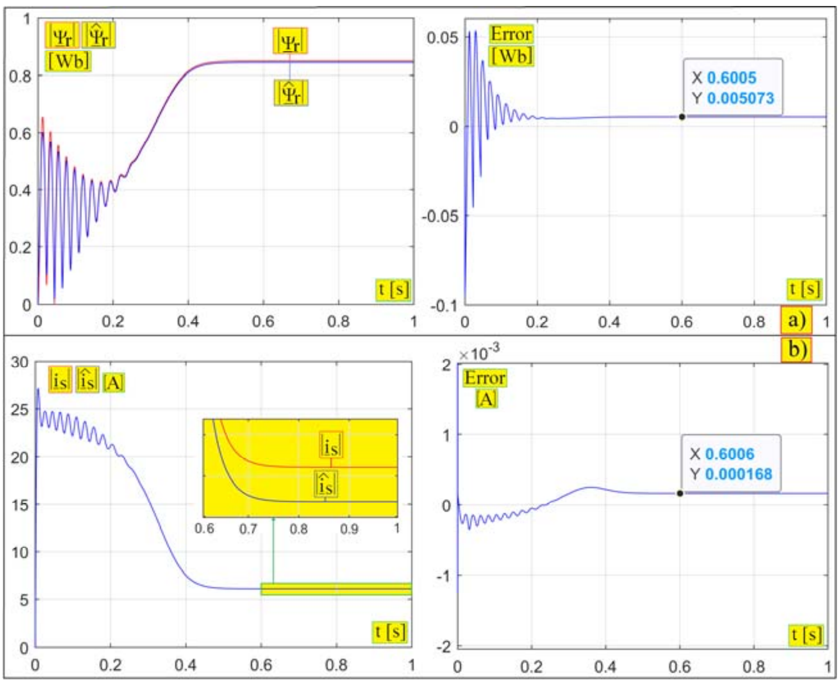

Fig.1 The time variation of the rotor flux space vector (a) and stator current space vector (b)

In the numerical simulation, formula (13) was used, in which: $k = 1.2$; $\theta = 30^o$. Based on the results in Fig. 1 we can say that the Luenberger observer estimates the state vector of the induction motor very well (the errors being very small). Tests have shown that the errors increase as the θ angle decreases. The optimal choice of the parameters k and θ in formula (13), as well as the study of the stability of the Luenberger observer, will be the subject of another article.

4. CONCLUSIONS

The article extends the field of research of the full-order state observers for the induction motors. The article presents a new gain matrix of the Luenberger observer.

Numerical simulation tests have shown that the new gain matrix of the observer allows the state vector estimation with very good errors. The Luenberger observer presented in this article successfully can be used in the vector control systems of induction motors.

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